Technical Notes

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Analysis of Open Noncircular Cylindrical Shells of Short Length

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Nomenclature

E = Young's modulus of elasticity
H = torsional moment
h = shell thickness
L,l = longitudinal length and lateral span
M_1,M_2 = bending moments in x and s directions
Q_1,Q_2 = normal shear forces corresponding to x and s directions
R = $R(\phi) = R(s)$ = radius of curvature of the profile
R_0 = $R(0)$ = nonzero, finite value of R at crown $s = 0$
r = 0, 1, 2, 3 = roots of the indicial equation
S = tangential shear force
s,x = natural coordinates along directrix and generator
T_1,T_2 = normal forces in x and s directions
u,v,w = displacement components in x, s and normal directions
$\Delta(\ )$ = $\partial^2(\ )/\partial\zeta^2 + \partial^2(\ )/\partial\eta^2$ = Laplace operator
η = s/R_0, $\zeta = x/R_0$ = nondimensional coordinates
λ_n = $n\pi R_0/L$, $\mu_n{}^2 = \lambda^2{}_n\cdot\mu^2$, $\mu^2 = R_0[12(I - \nu^2)]^{1/2}/h$
ν = Poisson's coefficient
ρ = $\rho(\eta) = R/R_0$ = transformed radius of curvature
Φ = $\Phi(\zeta,\eta)$ = stress function
ϕ = angle included between R_0 and R
$\tilde{\Psi}$ = $\tilde{\Psi}(\zeta,\eta)$ = complex stress-displacement function

Subscript

$(\ )_m$ = forces, displacement components with reference to homogeneous moment problem

Superscripts

$(\ )^0$ = forces, displacement components with reference to membrane problem
$(\ )^{(k)}$ = $\partial^k(\ )/\partial\eta^k$
$(\ )^*$ = $Re(\tilde{\ })$
$(\bar{\ })$ = $1m(\tilde{\ })$
$(\tilde{\ })$ = complex quantity

I. Introduction

NOVOZHILOV[1] and Vafakos, Romano, and Kempner[2] studied the problem of analysis of closed noncircular cylindrical shells. Grüning[3] developed a practical numerical method of integration of the governing equation of open noncircular cylindrical shells of short length in the complex form of Novozhilov.[1] In the present Note an analytical method of analysis of open noncircular cylindrical shells of short length simply supported at the curvilinear ends $\zeta = 0$ and $\zeta = L/R_0$ and having arbitrary boundary conditions along straight edges is developed. Accepting membrane solution of the cylindrical shell for the given surface loading as the particular solution[1,4] of the nonhomogeneous moment problem of the cylindrical shell, solution of the homogeneous moment problem (i.e., with no surface loadings) based on Donnell's approximations,[5] which, according to Gol'denveizer,[6] defines the state of stress with large indices of variation and nondegenerate edge effects, is constructed such that the total values of moments in the shell are directly obtained from the solution of the homogeneous moment problem, whereas the total values of forces and displacement components are given by: $T_1 = T_1{}^0 + T_{1m}$, $T_2 = T_2{}^0 + T_{2m}$, $S = S^0 + S_m$, $u = u^0 + u_m$, $v = v^0 + v_m$, $w = w^0 + w_m$.

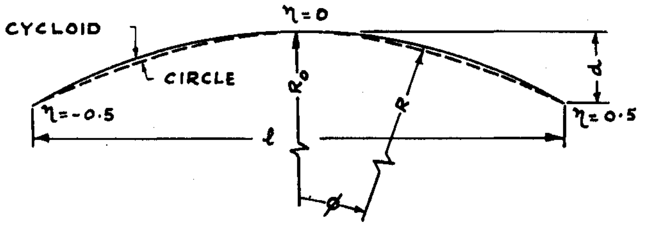

Fig. 1 Profiles of cycloidal and circular cylindrical shells.

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2. Governing Equations

The governing equations of the homogeneous moment problem of the cylindrical shells of short length based on the Donnell's approximations for the stress function Φ and normal displacement component w_m are given by[1,6]:

$$\Delta\Delta\Phi + (R_0Eh/\rho)(\partial^2 w_m/\partial\zeta^2) = 0$$
$$\Delta\Delta w_m - [12R_0(1 - \nu^2)/(Eh^3\cdot\rho)](\partial^2\Phi/\partial\zeta^2) = 0 \tag{1}$$

which are brought to a single equation for $\tilde{\Psi}$[1,6]:

$$\Delta\Delta\tilde{\Psi} + (i\mu^2/\rho)(\partial^2\tilde{\Psi}/\partial\zeta^2) = 0 \tag{2}$$

such that

$$\Phi = Re(\tilde{\Psi}),\ w_m = -(\mu^2/R_0Eh)Im(\tilde{\Psi}) \tag{3}$$

3. Solution

The solution of the Eq. (2) for the simply supported boundary conditions at $\zeta = 0$ and $\zeta = L/R_0$ is given by

$$\tilde{\Psi} = \sum_{n=1}^{\infty} \tilde{Y}_n(\eta)\sin(\lambda_n\zeta) \tag{4}$$

Then, from Eq. (2) the Normalform of Frobenius for $\tilde{Y}_n$ is obtained in the form[7]:

$$\eta^4(1)\tilde{Y}_n{}^{(4)} + \eta^2(-2\lambda_n{}^2\cdot\eta^2)\tilde{Y}_n{}^{(2)} + (\lambda_n{}^4\cdot\eta^4 - i\cdot\mu_n{}^2\cdot\eta^4/\rho)\tilde{Y}_n = 0 \tag{5}$$

For a class of regular and symmetrical profiles for which $R_0 \neq 0$ and $\neq \infty$ it has been derived[8] that

$$\rho = \sum_{k=0,2,4}^{\infty} \bar{a}_k\cdot\eta^k,\quad |\eta| < \eta_1;$$
$$\frac{1}{\rho} = \sum_{k=0,2,4}^{\infty} a_k\cdot\eta^k,\quad |\eta| < \eta_0 \leq \eta_1 \tag{6}$$

where coefficients $\bar{a}_k$ and a_k are to be determined for each profile, $\bar{a}_0 = a_0 = 1$. Then, about the regular point $\eta = 0$ Eq. (5) has 4 linearly independent analytic particular solutions of the form[7]:

$$\tilde{Y}_n = \eta^r \sum_{k=0}^{\infty} \tilde{g}_{kr}\cdot\eta^k = \sum_{k=0}^{\infty} \tilde{g}_{kr}\cdot\eta^{k+r}, \quad |\eta| < \eta_0 \qquad (7)$$

where index 'r' and complex coefficients $\tilde{g}_{kr}$ are unknown, η_0 is the radius of convergence of these series. Putting Eqs. (6) and (7) in Eq. (5), and then equating the coefficients of η^k to zero, it is found that for $\tilde{g}_{0r} \neq 0$ the roots of the indicial equation: $r(r-1)(r-2)(r-3) = 0$ are $r = 0, 1,2,3$ and all coefficients $\tilde{g}_{kr}(k = 2,4,6, \ldots)$ are recurrently expressed through $\tilde{g}_{0r} \neq 0$ for $r = 0,1,2,3$. Then, the general solution of Eq. (5) is given by[8]:

$$\tilde{Y}_n = \sum_{r=0}^{3}\left[\left(C_{(2r+1)n} + iC_{(2r+2)n}\right) \times \left(\sum_{k=0,2,4}^{\infty} g_{kr}^*\cdot\eta^{k+r} + i \sum_{k=4,6,8}^{\infty} \bar{g}_{kr}\cdot\eta^{k+r}\right)\right], \quad |\eta| < \eta_0 \qquad (8)$$

where $C_{(2r+j)n}$ are 8 constants of integration to be determined from the boundary conditions along straight edges, $g_{kr}^* = Re(\tilde{g}_{kr})$, $\bar{g}_{kr} = Im(\tilde{g}_{kr})$. Finally, from Eqs. (3) and (4) one obtains

$$\Phi = \sum_{n=1}^{\infty}\left[\sum_{r=0}^{3}\left(C_{(2r+1)n}\cdot Y_{nr}^* - C_{(2r+2)n}\cdot \bar{Y}_{nr}\right)\right]\cdot\sin(\lambda_n\zeta)$$
$$w_m = -\left(\frac{\mu^2}{R_0Eh}\right)\sum_{n=1}^{\infty}\left[\sum_{r=0}^{3}\left(C_{(2r+1)n}\cdot \bar{Y}_{nr} + C_{(2r+2)n}\cdot Y_{nr}^*\right)\right]\cdot\sin(\lambda_n\zeta) \qquad (9)$$

where

$$Y_{nr}^* = \sum_{k=0,2,4}^{\infty} g_{kr}^*\cdot\eta^{k+r}; \quad \bar{Y}_{nr} = \sum_{k=4,6,8}^{\infty} \bar{g}_{kr}\cdot\eta^{k+r}, \quad |\eta| < \eta_0$$

Then, all forces T_{1m}, T_{2m}, S_m are directly determined by differentiating Φ, while all moments M_1, M_2, H and Q_1,Q_2 by differentiating w_m. Using Hooke's law, u_m and V_m are determined.

4. Numerical Example

An open cylindrical shell with its profile in the form of a Cycloid, which is simply supported at its curvilinear ends $\zeta = 0$ and $\zeta = L/R_0$ and free of forces along straight edges $\eta = \pm 0.5$, has the following data: $R = R_0 \cos\phi$, $R_0 = 10$ m, $\rho = (1 - \eta^2)^{1/2}$, $\eta_0 = 1$, $L = 5\pi$ m, $l = 9.56611$ m, $d = 1.25$ m, (Fig. 1), $h = 0.06$ m, $\nu = 0$, $\lambda_1 = 2$. For the simply supported boundary conditions at $\zeta = 0$ and $\zeta = L/R_0$ the membrane solution of the cycloidal cylindrical shell under constant loading $q = 0.26$ t/m^2 $\approx$ $(4q/\pi)$ $\sin(\lambda_1\zeta)$ is accepted as the particular solution and then the solution (4) for the cycloidal cylindrical shell is given by $\tilde{\Psi} = \tilde{Y}_1(\eta)\cdot \sin(\lambda_1\zeta)$. As the problem is a symmetrical one, only two series $\tilde{Y}_{12}$ for $r = 0$ and $r = 2$ are calculated. Finally, the calculated values of forces and moments are compared with those in the circular cylindrical shell having the same controlling parameters and boundary conditions. Only the most interesting curves representing the maximum values of T_1 and M_2 in both the shells are shown here in Fig. 2 and Fig. 3.

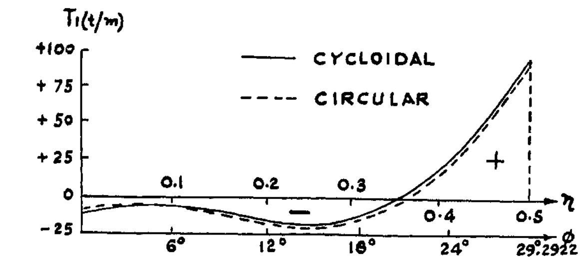

Fig. 2 Maximum values of normal force T_1 (tons/meter).

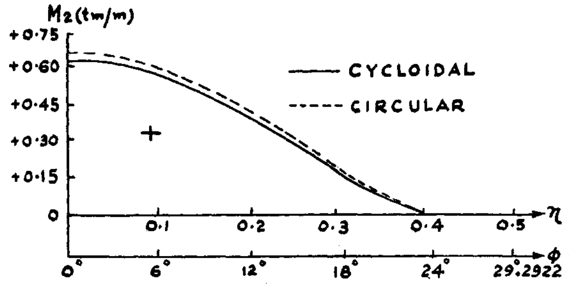

Fig. 3 Maximum values of bending moment M_2 (ton meter/meter).

5. Conclusions

The nature and character of distribution of the internal forces in both the shells must be identical, but their numerical values should negligibly differ from each other, as both the shells have identical controlling parameters and boundary conditions, and their profiles negligibly differ from each other. Comparison of the numerical results obtained has convincingly proved this inference, and thereby, conclusively established the correctness of the general method of analysis developed in the paper. In Fig. 3 it is interesting to observe that the curve of M_2 for cycloidal shell are throughout interior to these curve for circular shell, whereas the cycloidal arc is throughout exterior to the circular arc (Fig. 1). From these interesting results one may be tempted to conclude that open cylindrical shells with profiles having increasing curvature will be more economical than open circular cylindrical shells. But further investigations are essential, and may produce very interesting results of practical importance.

In the analysis of open noncircular cylindrical shells the governing equation (2) in complex form is recommended for use, as much less computer time will be involved than that required for Eqs. (1) in real form. The convergence of the power series of the solution is governed by the convergence of the series for $1/\rho$. Upper bounds of the truncation error of these power series of solution can be constructed with the help of majoring series. The solution obtained is very much suitable and convenient for computer and consequently, can find easy practical application.

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